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S. Y. B. Sc. (Sem.-III) Examination March - 2026

Mathematics - Major - MH -MJ-303 - Bivariate Calculus

[Total Marks: 50

સૂચના : / Instructions

(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the above details on your answer book

(2) All questions are compulsory.

(3) Figures to the right indicate marks of the corresponding question.

Seat No.:

Student's Signature

Q. 1. Answer the following (Any ten) **10**

1. Obtain f_z for $f(x, y, z) = \dfrac{x+y+z}{xy}$
2. If $x = u^2 + uv^2$ and $y = v^2 + u^2 v$ then find $J(x, y)$.
3. Define: Minima of a Bivariate function.
4. Obtain $\lim\limits_{x \to 0} \left\{ \lim\limits_{y \to 0} \dfrac{\sin(x+y)}{(x+y)} \right\}$
5. State the sufficient conditions for the extreme value of a bivariate function.
6. Evaluate $\int_1^2 \int_0^2 dx\, dy$.
7. Prove that $\lfloor n+1 = n \lfloor n$
8. Define: Homogeneous function.
9. Find the value of $\int_0^1 x^5(1-x)^4\, dx$.
10. Evaluate $\int_0^\infty x^5 e^{-x}\, dx$.
11. Define Extreme value of a Bivariate function.
12. Evaluate $\int_0^1 \int_0^{\frac{y}{2}} y\, dy\, dx$.

Q. 2. **Answer the following (Any two)** **10**

1. If $u = \log (x^3 + y^3 + z^3 - 3xyz)$, then prove that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 = \frac{9}{(x+y+z)^2}$
2. If f is a differentiable homogeneous function in x and y of degree m then prove that $x f_x + y f_y = mf(x,y)$
3. Find f_{xy} and f_{yx} for the function: $f(x, y) = \frac{x^2+y^2}{x+y}$, $x + y \neq 0$

Q. 3. **Answer the following (any two):** **10**

1. Find three positive real numbers whose sum is 15 and product is maximum.
2. Obtain expression of $f(x,y) = sin(ax + by)$ in terms of x and y.
3. If $x = \rho\, cos\phi$, $y = \rho\, sin\phi$, $z = z$ then find $\frac{\partial(x, y, z)}{\partial(\rho, \phi, z)}$

Q. 4. **Answer the following (Any two)** **10**

1. Evaluate $\iint_S (x + y^2)\, dx\, dy$ where $S = [0, 1] \times [x, \sqrt{x}\,]$.
2. Find the area enclosed between two parabolas $y^2 = 2x$ and $x^2 = 2y$.
3. Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} xy\, dx\, dy$ after changing the order of integration.

Q. 5. **Answer the following (Any two)** **10**

1. Prove the following:

   (i) $B(l, m) = \int_0^\infty \frac{y^{l-1}}{(1+y)^{l+m}}\, dy = \int_0^\infty \frac{y^{m-1}}{(1+y)^{l+m}}\, dy.$

   (ii) $\overline{|n} = \frac{1}{n} \int_0^\infty e^{-y^{\frac{1}{n}}}\, dy$
2. Evaluate $\int_0^3 \frac{dx}{\sqrt{3x - x^2}}$
3. State and prove the relation between Beta and Gamma function.